Open-source numerical simulation tool for two-dimensional neural fields involving finite axonal transmission speed

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Motivation

This work aims to provide an open source and cross-platform simulation tool that integrates numerically integral-differential equations of the type

$$\left(\eta\frac{\partial^2}{\partial t^2}+\gamma\frac{\partial}{\partial t}+1\right)V(x,t)=I(x,t)+\int_\Omega \mathrm{d}^2\mathrm{y}\,K(\parallel \mathrm{x}-\mathrm{y}\parallel)\,S\left[V\left(\mathrm{y},t-\frac{\parallel \mathrm{x}-\mathrm{y}\parallel}{c}\right)\right] \quad (1)$$

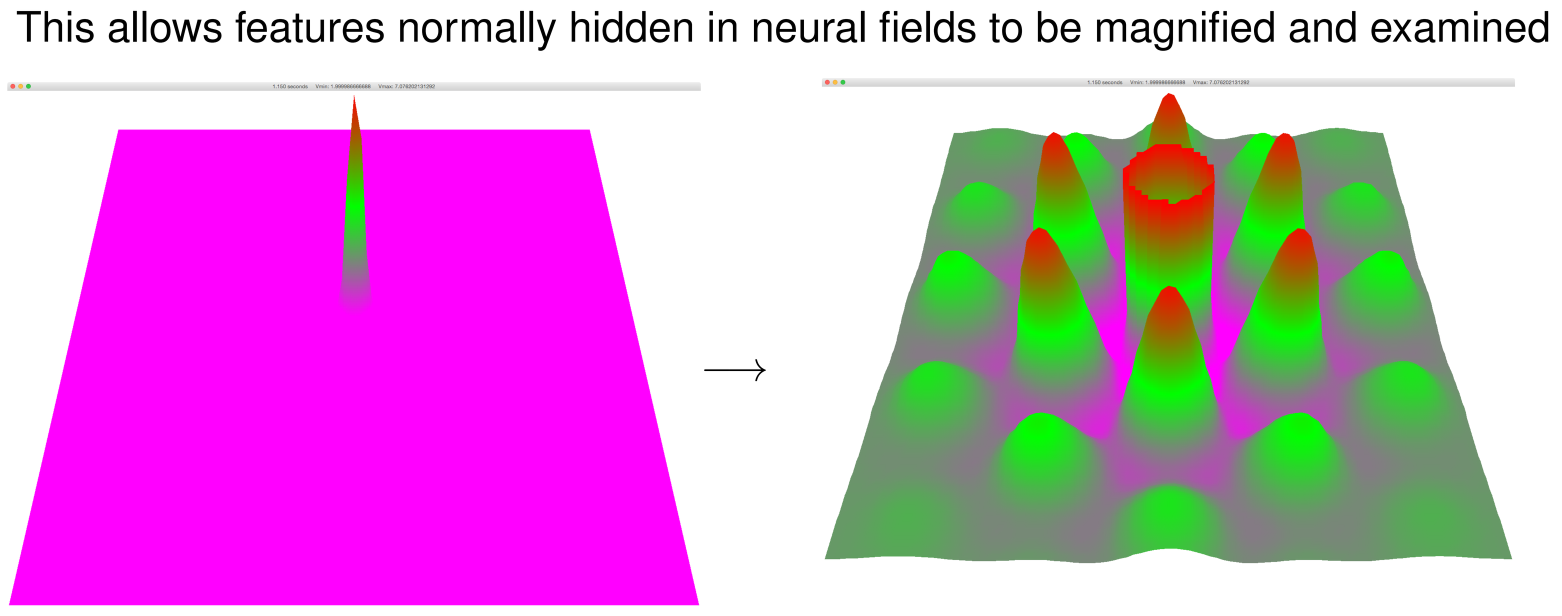
with mean neuron potential V in a two-dimensional quadratic spatial domain Ω with periodic boundary conditions. The term I denotes the external stimulus, K is the synaptic connectivity kernel and S is the firing rate. Finite axonal transmission speed c induces space-dependent delays.

To this end, we present the *Neural Field Simulator* [1].

Features

- ***Great usability***
  - Parametrization can be as simple or complex as field model
  - Visualization is easily modified by a keypress
- ***Complete control over Eq. (1) variables***
  - Free choice of values provided by text-based Python interface
- ***Spatio-temporal kernel***
  - Integral renders into a spatial integral and an integral over delays [2]
- ***Optimal acceleration***
  1. Fast Fourier transform in space
  2. Self-writing code based on interface selections
  3. Utilization of graphics processing unit for hardware acceleration
  4. Reduced rate of GPU uploads optimised for visual perception
- ***Output in rich detail***
  - 2D matrices output in 3D whereby $[x,y]\mapsto[z]$

  This allows features normally hidden in neural fields to be magnified and examined



  - Matrices can be moved, rotated, zoomed and colors and axis limits are easily changed
  - Movies and images of simulations can be saved

Breather

A breather (Fig. 1) is simulated with a spatial grid of $n{=}512$ squared elements and parameters $dt{=}0.001$, $\gamma{=}1$, $\eta{=}0$, length $l{=}30$, $V(t{=}0){=}0$, $V_{noise}(t{>}0){=}\frac{e^{(a^2+b^2)}\sqrt{\partial t}}{320\pi}$ where a and b are a meshgrid of $[-l/2,\cdots,l/2]$, $I{=}\frac{20e^{-x^2/32}}{32\pi}$, $K{=}\frac{-e^{-x/3}}{4.5\pi}$, $S{=}\frac{1}{1+e^{-10000(V-0.005)}}$ and $c{=}500$.

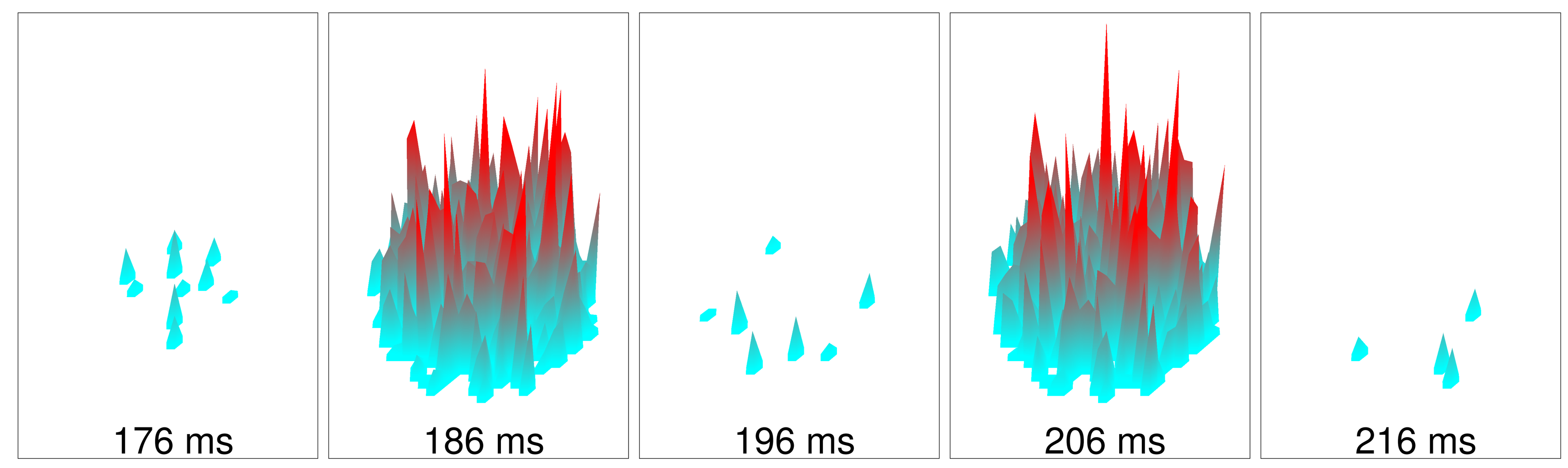


Figure 1: A breather at 10 millisecond intervals with manually set minimum Z axis (=0.0048).

Turing Pattern

Turing patterns emerge from noisy field voltage (Fig. 2) with terms $dt{=}0.01$, $\gamma{=}1$, $\eta{=}0$, $l{=}90$, $n{=}512$, $V(t{=}0){=}5.4+\frac{e^{-x^2/0.02}}{\sqrt{0.02\pi}}$, $I{=}0$, $K{=}K_{\_}-\sin(\vec{v})/200$ with $K_{\_}{=}\sin(\vec{v})/150$ and $\vec{v}{=}[-9\pi,\ldots,0]$, $S{=}\frac{2}{1+e^{-1.24(V-3)}}$ and $c{\geq}l\sqrt{2}\Delta t$ is infinite (=6364).

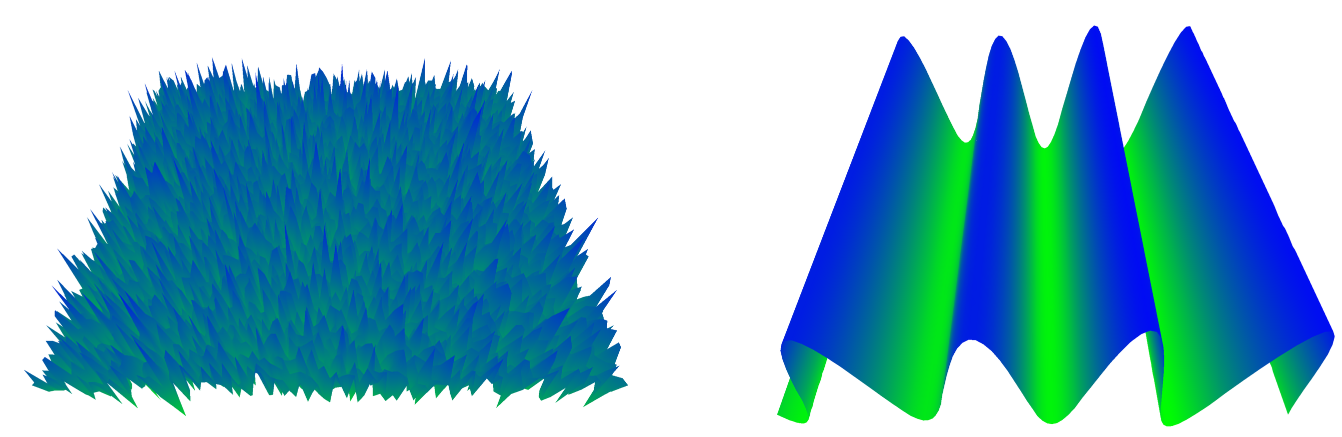


Figure 2: A smooth constant Turing pattern exists from noise after 5 seconds.

Alternating Roll

Reference [1] presents an alternating roll solution (Fig. 3) with descriptions of the following values: $dt{=}0.02$, $\gamma{=}2$, $\eta{=}1$, $l{=}40.3805226$, $n{=}512$, $k_c{=}1.0891958379832$ $\omega_c{=}3.4003003526352$ $V(t{=}0){=}3+0.4\sin(k_c a)$, $U_{excite}{=}0.4\omega_c\cos(k_c b)$ $I{=}2.5$, $K{=}\frac{121e^{-x}-235.2e^{-1.4x}}{2\pi}$, $S{=}\frac{1}{1+e^{-2.856(V-3)}}$ and finite $c{=}6$.

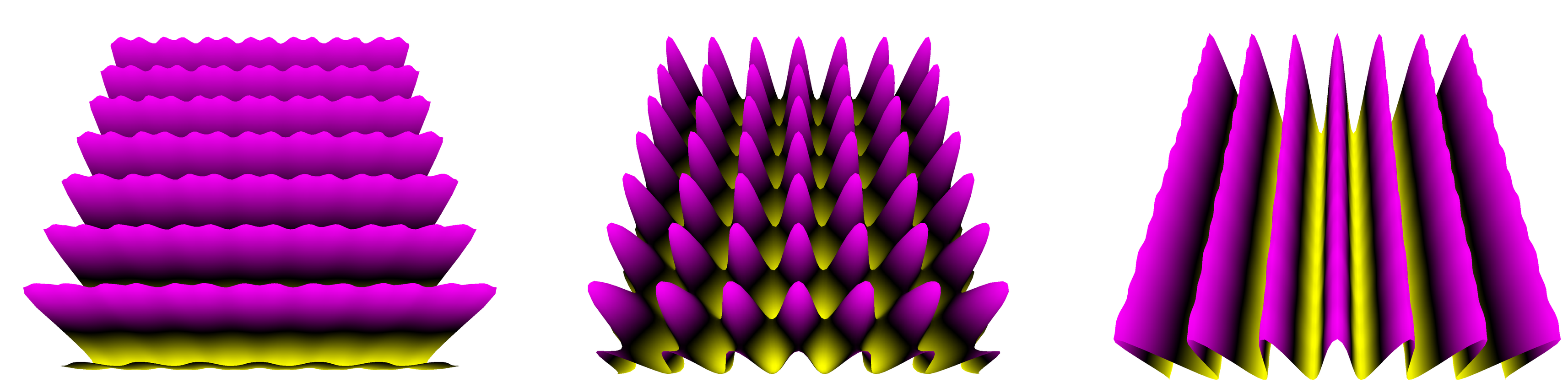


Figure 3: A stable alternating roll continuously transforms between horizontal and vertical line patterns every ≈31 milliseconds.

Acknowledgments

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References

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